

NEUTRINOS: “...ANNUS MIRABILIS”

A. YU. SMIRNOV

*International Centre for Theoretical Physics, Strada Costiera 11,
31014 Trieste, Italy, and
Institute for Nuclear Research, Russian Academy of Sciences, Moscow, Russia
E-mail: smirnov@ictp.trieste.it*

ABSTRACT

Main results and achievements of 2002 - 2003 in neutrino physics are summarized. The field moves quickly to new phase with clear experimental and phenomenological programs, and probably, with new theoretical puzzle which may lead us to discoveries of the fundamental importance. One of the main results is amazing pattern of the lepton mixing which emerges from the data. The key questions are: Does lepton mixing imply new symmetry of Nature? Is the large (maximal?) mixing related to degeneracy of the neutrino mass spectrum? In this connection priorities of the future studies are formulated.

1. One year after

Year 2002 started by the SNO publication of the direct evidence for the solar neutrino flavor conversion¹⁾ and finished by an announcement of the first KamLAND result²⁾ has been called in Ref.³⁾ the “*annus mirabilis*” of the solar neutrino physics. In 2002 the pioneering works on the detection of solar neutrinos have gotten the highest appreciation and in the same year the solar neutrino problem (which was the outcome of this detection and the driving force of developments in neutrino physics during last 35 years) has been essentially resolved. The beginning and the end have met. What happened after? What is an impact of the *annus mirabilis* on the field?

From the scientific calendar starting 1 year back from now^a:

- *December 4, 2002.* K2K⁴⁾: “Indication of neutrino oscillations in a 250 km long baseline experiment”.
- *December 6, 2002.* KamLAND²⁾: “First Results from KamLAND: Evidence for Reactor for reactor anti-neutrino disappearance”.
- *December 10, 2002.* The ceremony of the Nobel Prize award: R. Davis Jr. and M. Koshiba: “... for pioneering contribution to astrophysics, in particular for the detection of cosmic neutrinos”.
- *February 11, 2003.* WMAP⁵⁾: “First year Wilkinson microwave anisotropy probe observations: determination of cosmological parameters...”

^aTalk given at the 2nd Int. Workshop on Neutrino oscillations in Venice (NOVE) December 3-5, 2003, Venice, Italy

- *September 3, 2003.* SuperKamiokande-I⁶⁾: “Precise measurement of the solar neutrino Day/Night and seasonal variations in Super-Kamiokande-I.
- *September 7, 2003.* SNO salt phase results⁷⁾: “Measurements of the total active B-8 solar neutrino flux at the Sudbury neutrino observatory with enhanced neutral current sensitivity”.
- *October 27, 2003.* Sloan Digital Sky Survey⁸⁾: “Cosmological parameters from SDSS and WMAP”.

There is a number of immediate consequences of these results:

- 1). The LMA MSW solution of the solar neutrino problem is confirmed.
- 2). The oscillation parameters Δm_{12}^2 and θ_{12} are determined with reasonable accuracy. In particular, significant deviation of the 1-2 mixing from maximal is established.
- 3). The key step is done in the reconstruction of the neutrino mass and flavor spectrum. The dominant structures of the mixing matrix and both Δm^2 (apart from the sign of Δm_{13}^2) are known.
- 4). The confirmation of the LMA solution opens a possibility to measure the CP violation in the leptonic sector in the future LBL experiments.
- 5). In the connection to LMA, a possibility of substantial cancellation of contributions in the neutrinoless double beta decay is confirmed. This, in turn, has serious impact on perspectives of determination of the absolute scale of neutrino mass and the role of the Majorana phases.
- 6). Picture of the flavor conversion of neutrinos from SN 1987A is determined.
- 7). The LMA oscillations of the atmospheric neutrinos should exist. This opens new possibility to search for the deviation of 2-3 mixing from maximal.
- 8). Strong bound on the leptonic asymmetry of the Universe is established.
- 9). The first KamLAND result marks the birth of neutrino geophysics.
- 10). Important cosmological bound on neutrino mass is given.

These results moved the field to new phase with new goals, experimental programs and theoretical problems.

2. Summarizing achievements

2.1. After SNO salt results

The SNO salt phase results⁷⁾ have further confirmed the correctness of the Standard Solar Model (SSM) neutrino fluxes⁹⁾ and the realization of the MSW large mixing (LMA) conversion mechanism¹⁰⁾ inside the Sun.^{11)–17)} In Fig. 1 we show

the allowed region of the oscillation parameters $\tan^2 \theta_{12}$ and Δm_{12}^2 from the 2ν analysis of the solar neutrino data (left) and from the combined analysis of the solar neutrino and KamLAND²⁾ results (right). The best-fit values of the parameters are

$$\Delta m_{12}^2 = 7.1 \times 10^{-5} \text{eV}^2, \quad \tan^2 \theta_{12} = 0.4. \quad (1)$$

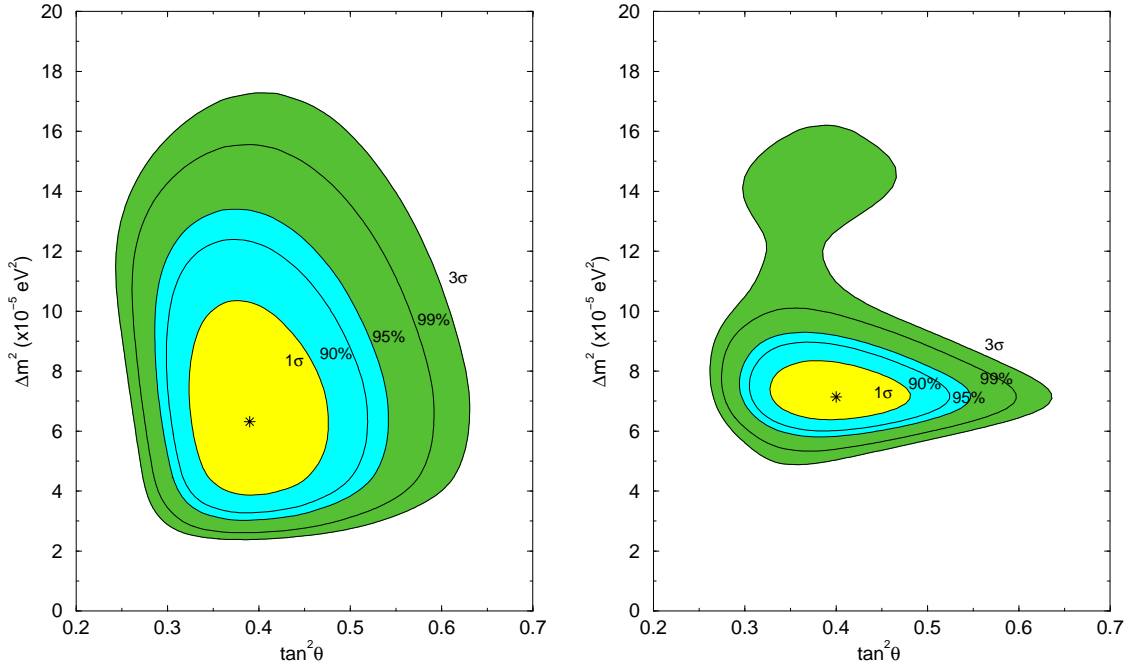


Figure 1: The allowed regions of oscillation parameters from the 2ν analysis of the solar neutrino data (left) and the combined fit of the solar neutrino data and the KamLAND spectrum (right) at 1σ , 2σ , 3σ CL.¹⁵⁾

Combined fit of the solar, KamLAND²⁾ and CHOOZ¹⁸⁾ results favors nearly zero 1-3 mixing: $\sin^2 \theta_{13} \sim 0$.^{16,17)} Implications of these results can be formulated in the following way.

- 1). The l-LMA region with $\Delta m_{12}^2 < 10^{-4} \text{eV}^2$ is selected, and the h-LMA region is accepted now at 3σ only.
- 2). Maximal 1-2 mixing is strongly disfavored. The upper bound is

$$\tan^2 \theta_{12} < 0.64 \quad (3\sigma). \quad (2)$$

That is, significant deviation of the 1 - 2 mixing from maximal is established which can be expressed as

$$1/2 - \sin^2 \theta_{12} \sim \sin^2 \theta_{12}. \quad (3)$$

3). As a result of more precise determination of the oscillation parameters the physics of the conversion is now determined quantitatively.^{12,17)} In particular, recent results show relevance of the notion of resonance, they fix the relative strength of the effects of the adiabatic conversion and the oscillations as function of the neutrino energy¹⁷⁾.

Next KamLAND data release is extremely important for understanding stability of the results, backgrounds, contribution of the geo neutrinos and more precise determination of parameters.

Concerning potential problems of the LMA solution, namely, the low Homestake rate¹⁹⁾ and the absence of the upturn of the boron neutrino spectrum at low energies: Recent measurements of the nuclear cross-sections by LUNA experiment lead to decrease of the CNO fluxes²⁰⁾, and consequently, reduced difference of the Homestake result and the LMA prediction²¹⁾. Forthcoming SNO spectral results may shed some light on existence of the upturn²¹⁾.

2.2. Atmospheric neutrinos and 2-3 mixing

A recent refined analysis of the SuperKamiokande data in terms of $\nu_\mu - \nu_\tau$ oscillations gives²²⁾ at 90 % C.L.

$$\Delta m_{13}^2 = (1.3 - 3.0) \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{23} > 0.91 \quad (4)$$

with the best fit at $\Delta m_{12}^2 = 2.0 \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta_{12} = 1.0$. Combined analysis of the CHOOZ and the atmospheric neutrino data puts the upper bound on the 1-3 mixing²³⁾

$$\sin^2 \theta_{13} < 0.067 \quad (3\sigma). \quad (5)$$

The open question is whether oscillations of the atmospheric ν_e exist? There are two possible sources of these oscillations: (i) non-zero 1-3 mixing and “atmospheric” Δm_{13}^2 , and (ii) solar oscillation parameters in Eq. (1). Also their interference should exist.²⁴⁾ After confirmation of the LMA-MSW solution we can definitely say that oscillations driven by the LMA parameters (the LMA oscillations) should show up at some level. Relative change of the ν_e flux due to the LMA oscillations can be written as²⁴⁾

$$\frac{F_e}{F_e^0} - 1 = P_2(r \cos^2 \theta_{23} - 1), \quad (6)$$

where $P_2(\Delta m_{12}^2, \theta_{12})$ is the 2ν transition probability and $r \equiv F_\mu^0/F_e^0$ is the ratio of the original ν_μ and ν_e fluxes. In the sub-GeV region, where P_2 can be of the order 1, the ratio equals $r \approx 2$, so that the oscillation effect is proportional to the deviation of the 2-3 mixing from the maximal value:

$$D_{23} \equiv 1/2 - \sin^2 \theta_{23} \quad (7)$$

In Fig. 2 from ²⁴⁾ we show the ratio of numbers of the e -like events with and without oscillations as function of the zenith angle of the electron. For the allowed range of $\sin^2 \theta_{23}$ and the present best-fit value of Δm_{12}^2 the excess can be as large as 5 - 6%. The excess increases with decreasing energy.

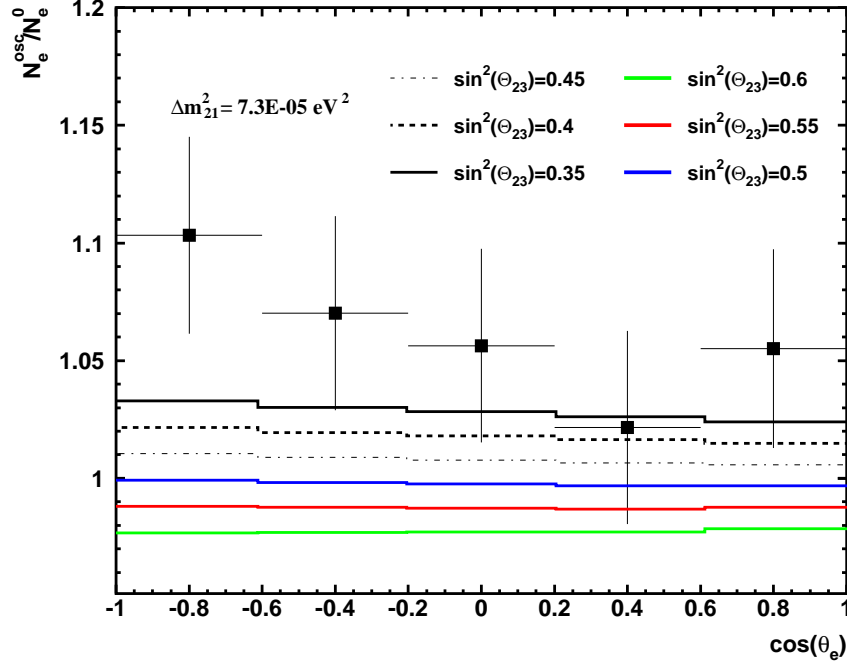


Figure 2: The ratio of numbers of the e -like events with and without oscillations as function of the zenith angle of the electron for different values of $\sin^2 \theta_{23}$.²²⁾ Other parameters are $\sin^2 2\theta_{12} = 0.82$, $\sin \theta_{13} = 0$ and $\Delta m_{12}^2 = 7.3 \times 10^{-5} \text{ eV}^2$. Also shown are the SuperKamiokande experimental points.

Future searches for the excess can be used to restrict or measure D_{23} . In fact, the latest analysis, (without renormalization of the original fluxes) shows some excess of the e -like events at low energies and the absence of excess in the multi-GeV sample, thus giving a hint of non-zero D_{23} . Establishing this deviation has important consequences for understanding the origins of neutrino masses and mixing.

In Fig. 3 we show the contours of constant excess of the e -like events for zero 1-3 mixing. According to this figure, establishing the excess at the level 3% would imply for $\Delta m_{12}^2 = 7 \cdot 10^{-5} \text{ eV}^2$ the lower bound on the deviation: $D_{23} > 0.17$. The 1-3 mixing generates the interference effect between the LMA oscillations amplitudes.²⁴⁾ The interference contribution does not contain the “screening” factor, as in Eq. (6), and can reach 2–4% for the allowed values of $\sin \theta_{13}$. This produces an uncertainty in the determination of D_{23} which can be reduced once stronger bound on 1-3 mixing is obtained. In any case observation of the excess of e -like events at the level of $\sim 5\%$

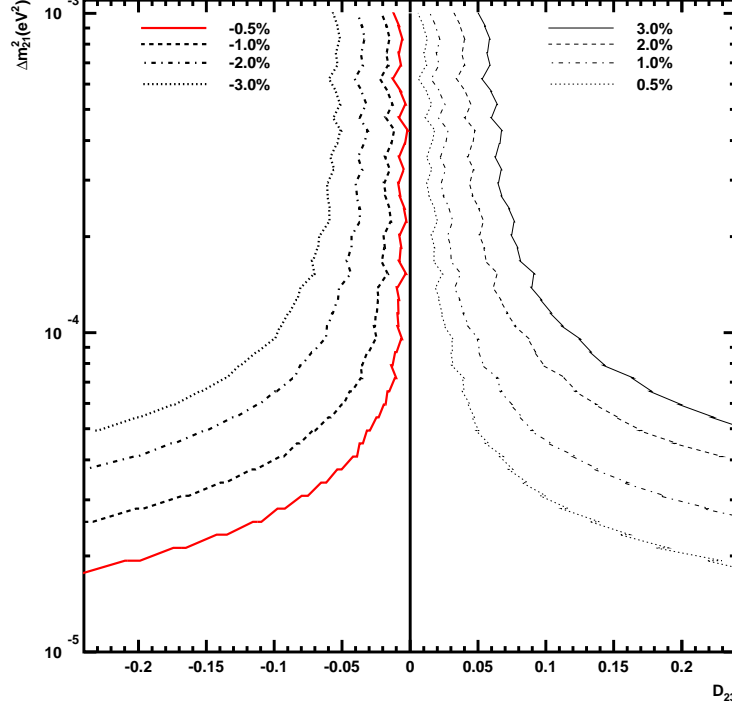


Figure 3: Contours of constant excess of the e - like events in the Δm_{12}^2 - D_{23} plane.²²⁾

will imply strong deviation of the 2-3 mixing from the maximal one.

2.3. Neutrinos from SN1987A: flavor conversion

After confirmation of the LMA-MSW solution we can definitely say that the effect of flavor conversion has already been observed in 1987. One must take into account the conversion effects in analysis of SN1987A²⁵⁾ and future supernova neutrino data.

In terms of the original fluxes of the electron, $F^0(\bar{\nu}_e)$, and muon, $F^0(\bar{\nu}_\mu)$, antineutrinos, the electron antineutrino flux at the detector can be written as

$$F(\bar{\nu}_e) = F^0(\bar{\nu}_e) + \bar{p}\Delta F^0, \quad (8)$$

where $\Delta F^0 \equiv F(\bar{\nu}_\mu) - F(\bar{\nu}_e)$, and \bar{p} is the permutation factor. In assumptions of the normal mass hierarchy (ordering) and the absence of new neutrino states, \bar{p} can be calculated precisely: $\bar{p} = 1 - P_{1e}$, where P_{1e} is the probability of $\bar{\nu}_1 \rightarrow \bar{\nu}_e$ transition inside the Earth.^{26,27)} It can be written as $\bar{p} = \sin^2 \theta_{12} - f_{reg}$, where f_{reg} describes the effect of oscillations (regeneration of the $\bar{\nu}_e$ flux) inside the Earth. Due to the difference in distances traveled by neutrinos to Kamiokande, IMB and Baksan detectors

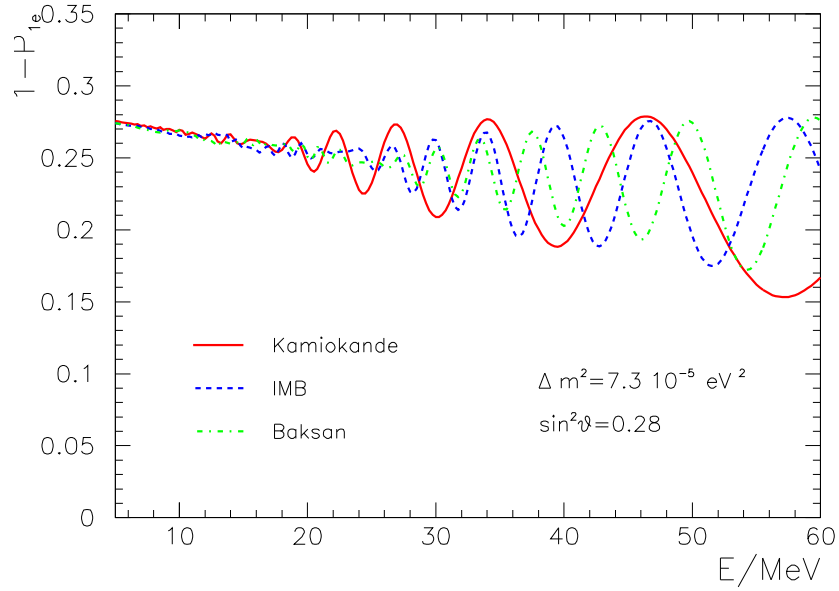


Figure 4: The permutation factor $\bar{p} = 1 - P_{1e}$ as a function of neutrino energy for Kamiokande II, IMB and Baksan detectors.²⁶⁾

inside the Earth: 4363 km, 8535 km and 10449 km correspondingly, the permutation factors differ for these detectors (Fig: 4). The Earth matter effect can partially explain the difference between the Kamiokande and the IMB spectra of events.²⁷⁾

In contrast to \bar{p} , the effect of conversion on the observed signals depends substantially on the parameters of original neutrino spectra. As an illustration, in Fig: 5 we show the average energy of the observed events as a function the average energy of the original $\bar{\nu}_e$ spectrum E_{0e} for different values of $r_E \equiv E_{0\mu}/E_{0e}$ and $r_L \equiv L_{0\mu}/L_{0e}$ for Kamioka-2 (upper panel) and IMB (lower panel)²⁸⁾. Notice that conversion can lead to increase of the average energy by (30 - 40)%. Inversely, as follows from Fig: 5, not taking into account the conversion can lead to errors in determination of the average energy of original spectrum of the order 40 - 50 % in K2, and factor of 2 in IMB.

For the inverted mass hierarchy and $\sin^2 \theta_{13} > 10^{-4}$ one would get nearly complete permutation, $\bar{p} \approx 1$, and therefore a harder $\bar{\nu}_e$ spectrum, as well as the absence of the Earth matter effect. This is disfavored by the data,²⁹⁾ though in view of small statistics and uncertainties in the original fluxes it is not possible to make a firm statement.

2.4. Absolute Scale of Mass

From the oscillation results we can put a lower limit on the heaviest neutrino

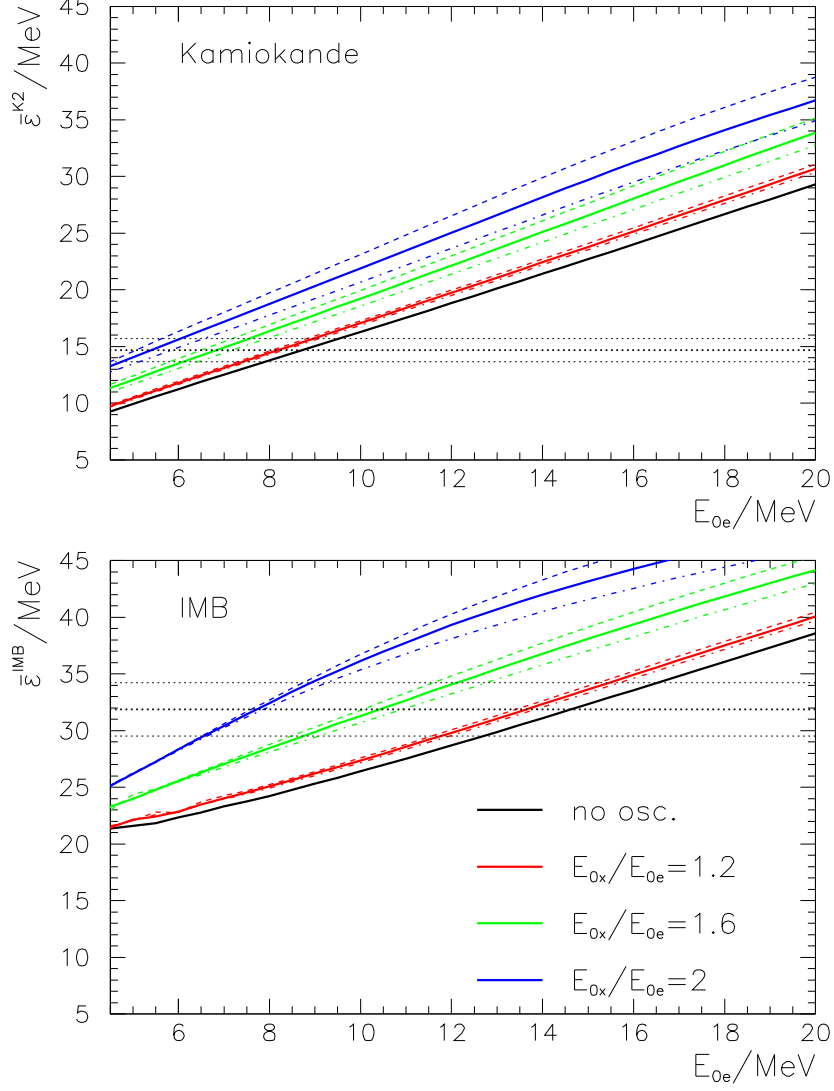


Figure 5: The dependence of the average energy of the observed events on the average energy of the original $\bar{\nu}_e$ spectrum E_{0e} for different values of $r_E \equiv E_{0\mu}/E_{0e}$ and $r_L \equiv L_{0\mu}/L_{0e}$ for Kamioka-2 (upper panel) and IMB (lower panel)²²⁾.

mass:

$$m_h \geq \sqrt{\Delta m_{13}^2} > 0.04 \text{ eV}, \quad (9)$$

where $m_h = m_3$ for the normal mass hierarchy, and $m_h = m_1 \approx m_2$ for the inverted hierarchy. The neutrinoless double beta decay is determined by the effective mass

$$m_{ee} = \left| \sum_k U_{ek}^2 m_k e^{i\phi(k)} \right|, \quad (10)$$

where $\phi(k)$ is the Majorana phase of the k eigenvalue. Fig: 6 from ³⁰⁾ summarizes the present knowledge of the absolute mass scale. Shown are the allowed (at 90 % CL) regions in the plane of m_{ee} probed by the $\beta\beta_{0\nu}$ decay and the mass of lightest neutrino probed by the direct kinematical methods and cosmology. The best present bound on m_{ee} is given by the Heidelberg-Moscow experiment: $m_{ee} < 0.35 - 0.50$ eV,³¹⁾ part of collaboration claims an evidence of a positive signal.³²⁾

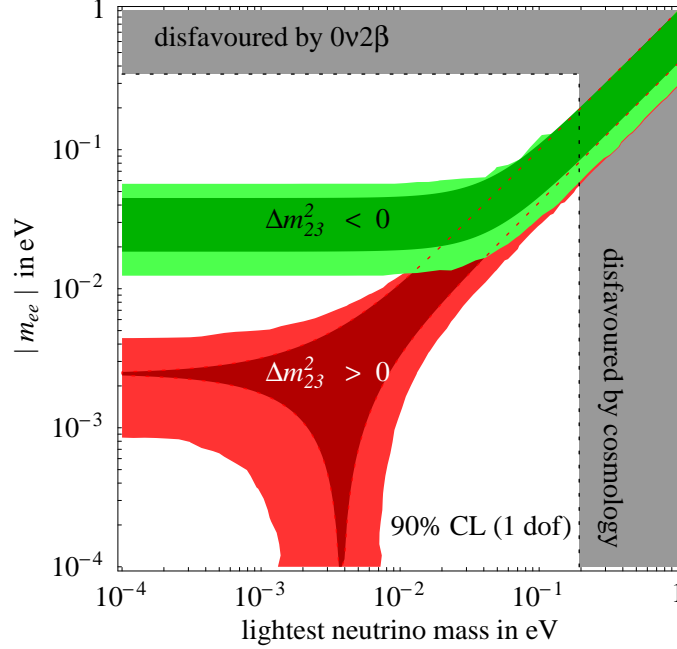


Figure 6: The 90% CL range for m_{ee} as a function of the lightest neutrino mass for the normal ($\Delta m_{23}^2 > 0$) and inverted ($\Delta m_{23}^2 < 0$) mass hierarchies ²⁸⁾. The darker regions show how the allowed range for the present best-fit values of the parameters with negligible errors.

The present double beta decay searches and cosmology have similar sensitivities: $m_{ee} \sim m_1 \sim (0.2 - 0.5)$ eV. This value corresponds to the degenerate mass spectrum: $m_1 \approx m_2 \approx m_3 \equiv m_0$. Analyses of cosmological data (with WMAP) result in the 95% C.L. upper bounds $m_0 < 0.23$ eV,³³⁾ $m_0 < 0.6$ eV³⁴⁾ and $m_0 < 0.34$ eV.³⁵⁾ Independent analysis which includes the X-ray galaxy cluster data gives non-zero value $m_0 = 0.20 \pm 0.10$ eV.³⁶⁾

Future improvements of the upper bound on m_{ee} have the potential to distinguish between the hierarchies: if the bound $m_{ee} < 0.01$ eV is established, the inverted hierarchy will be excluded at 90 % C.L. (see Fig: 6).

2.5. Mass Spectrum and Mixing

Information obtained from the oscillation experiments allows us to make signifi-

cant progress in the reconstruction of the neutrino mass and flavor spectrum (Fig: 7).

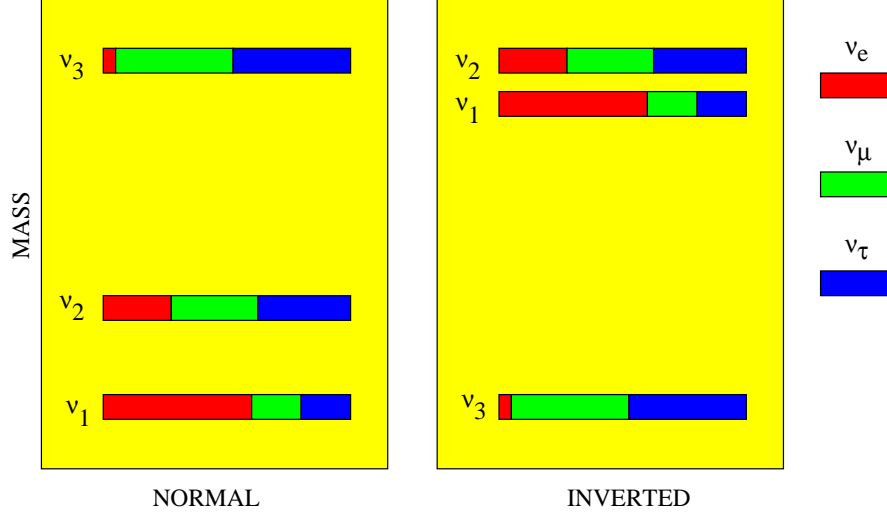


Figure 7: Neutrino mass and flavor spectra for the normal (left) and inverted (right) mass hierarchies. The distribution of flavors (colored parts of boxes) in the mass eigenstates corresponds to the best-fit values of mixing parameters and $\sin^2 \theta_{13} = 0.05$.

Using a global fit of the oscillation data one can find ³⁷⁾ the (90% CL) intervals for the elements of the PMNS mixing matrix $||U_{\alpha i}||$:

$$\begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.0 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}, \quad (11)$$

where columns correspond to the flavor index and rows to the mass index.

Now we are in a position to construct the leptonic unitarity triangle, though the finite size of one angle and therefore the length of one is still unknown. For practical reason (no intensive ν_τ beams) we consider the triangle which employs the e - and μ -rows of the mixing matrix (Fig: 8). The triangle is not degenerate in spite of the strong bound on the 1-3 mixing.

The area of the triangle is related to the Jarlskog invariant $J_{CP} \equiv \text{Im}[U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*]$: $S = J_{CP}/2$. Reconstruction of the triangle is complementary to measurements of the neutrino-antineutrino asymmetries in oscillations. The main problem here is the coherence. For the triangle method we need to study interactions of the mass eigenstates, whereas in practice we deal with flavor (coherent) states. So, breaking of the coherence, averaging of oscillations, experiments with the beams of mass eigenstates and measurements of the survival (rather than transition) probabilities are the key elements of the method.³⁸⁾

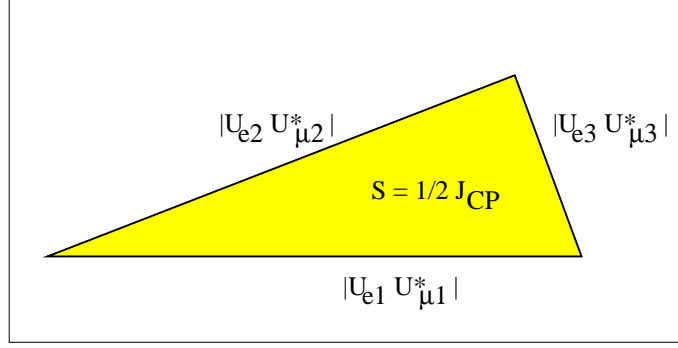


Figure 8: Possible leptonic unitarity triangle. We take the best-fit values of θ_{12} , and θ_{23} and $\sin \theta_{13} = 0.16$.

2.6. Main features

Information on neutrino masses and mixing can be summarized in the following way.

1). Taking the lower bound from Eq.(9) and the upper cosmological bound we can conclude that the absolute scale of neutrino mass is known within one order of magnitude:

$$m_h \sim 0.04 - 0.4 \text{ eV}. \quad (12)$$

This interval is still too large from the theoretical point of view. Depending on specific value of the mass within this interval, one arrives at completely different conclusions.

2). The observed ratio of the mass squared differences, $\Delta m_{12}^2 / \Delta m_{23}^2 = 0.01 - 0.15$, implies that there is no strong hierarchy of neutrino masses:

$$\frac{m_2}{m_3} > \sqrt{\frac{\Delta m_{12}^2}{\Delta m_{23}^2}} = 0.18_{-0.08}^{+0.22}. \quad (13)$$

For charge leptons the corresponding ratio is 0.06.

3). There is the bi-large or large-maximal mixing between the neighboring families (1 - 2) and (2 - 3). Still rather significant deviation of the 2-3 mixing from the maximal one is possible and it is not excluded, *e.g.*, that 1-2 and 2-3 are equal. Mixing between remote (1-3) families is weak.

There is interesting and rather precise relation

$$\theta_{12} + \theta_c = \theta_{23} \sim 45^\circ, \quad (14)$$

where θ_c is the Cabibbo angle. It is not clear if this equality is just accidental coincidence. It seems there is no simple scenario which leads to the equality $\theta_{12} =$

$45^\circ - \theta_c$, that is, to scenario in which the lepton mixing = bi-maximal mixing (which follows from the neutrino mass matrix) - corrections (which follow from the charge lepton mass matrix). Though there is again an unexpected relation

$$\tan \theta_c \approx \sqrt{\frac{m_\mu}{m_\tau}} = \lambda \approx 0.2. \quad (15)$$

This may further testify for quark - lepton similarity.

2.7. Achieved results: where we are?

The achieved results allow us to formulate clear program of further phenomenological and experimental studies which includes determination of

- the absolute mass scale m_1 ;
- the type of mass spectrum; hierarchical; non-hierarchical with certain ordering; degenerate, which is related to the value of m_1 ;
- the type of mass hierarchy (ordering): normal, inverted;
- the 1-3 mixing;
- the CP-violating Majorana phases;
- deviations of the 2-3 from maximal.

An important issue is searches for new neutrino states.

On the other hand we get new theoretical puzzle which may lead us to new fundamental discoveries. The puzzle is related to unexpected pattern of the lepton mixing and possible mass spectrum.

3. Toward the underlying physics

The first step in attempts to uncover the underlying physics could be reconstruction of the neutrino mass matrix and studies of its properties. Here we assume that neutrinos are the Majorana particles.

3.1. Neutrino mass matrix

The Majorana mass matrix of neutrinos in the flavor basis can be written as

$$m = U^* m^{diag} U^+, \quad (16)$$

where $U = U(\theta_{ij}, \delta)$ is the mixing matrix, δ is the Dirac CP-violating phase, and

$$m^{diag} \equiv diag(m_1 e^{-2i\rho}, m_2, m_3 e^{-2i\sigma}). \quad (17)$$

Here ρ and σ are the Majorana phases. The mass eigenvalues equal $m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}$, and $m_3 = \sqrt{m_1^2 + \Delta m_{13}^2}$.

The results of reconstruction of the mass matrix^{39,40)} are shown in Figs: 9, 10, and 11 as the $\rho - \sigma$ plots for the absolute values of the 6 independent matrix elements⁴⁰⁾. These figures correspond to three extreme cases: normal mass hierarchy, quasi-degenerate spectrum and inverted mass hierarchy. The figures illustrate a variety of possible structures. In particular, for the normal mass hierarchy (Fig: 9) there is clear structure with the dominant $\mu - \tau$ block. Interesting parameterizations of the mass matrix (up to an overall mass factor) are

$$\begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad (18)$$

where $\lambda \sim 0.2$. Also the matrix similar to the first one in Eq. (18) with $m_{12} \sim \lambda$ and $m_{13} \approx 0$ is possible.

In the case of a quasi-degenerate spectrum, the interesting dominant structures are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (19)$$

These matrices are realized for values of phases in the corners of the plots: $\rho, \sigma = 0, \pi$ (the first matrix) or at $\rho = 0, \pi, \sigma = \pi/2$ (the second one) which corresponds to definite CP-parities of the mass eigenstates. Also the “democratic” structure with equal moduli of elements is possible for the non-trivial values of phases.⁴¹⁾ Changing the phases one can get any intermediate structure between those in Eqs. (18) and (19).

In the case of the inverted hierarchy the ee - element is not small generically. Among interesting (and very different) examples are

$$\begin{pmatrix} 0.7 & 1 & 1 \\ 1 & 0.1 & 0.1 \\ 1 & 0.1 & 0.1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 0.5 & 0.5 \\ 0.1 & 0.5 & 0.5 \end{pmatrix}. \quad (20)$$

Notice that there are clear advantages to consider the mass matrix instead of oscillation parameters:

1). The mass matrix unifies information contained in masses and mixing angles and this may provide some deeper insight to the underlying theory.

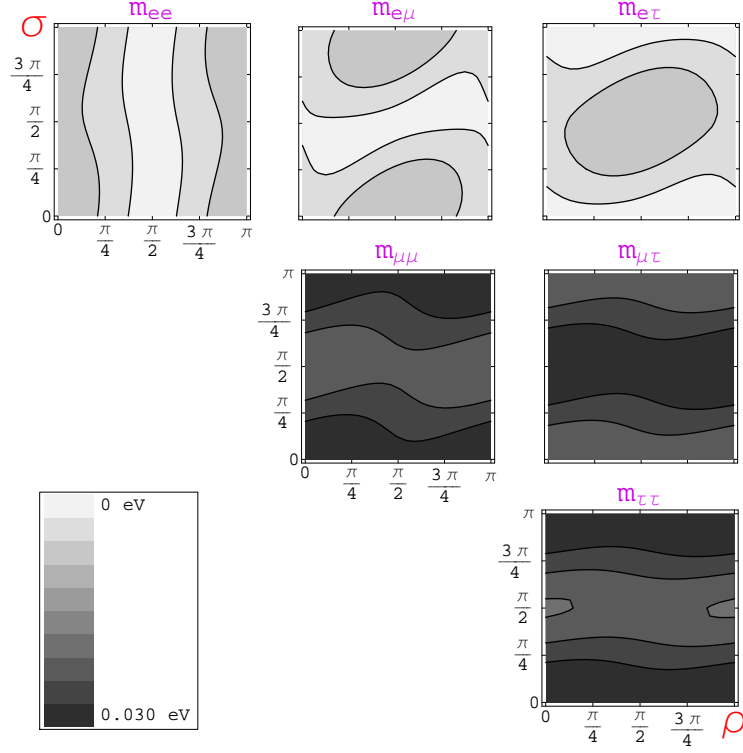


Figure 9: The Majorana mass matrix for the normal mass hierarchy: $m_3/m_2 = 5$, $m_1 \approx 0.006$ eV. We show contours of constant mass in the $\rho - \sigma$ plots for the moduli of mass matrix elements. We take for other parameters $\Delta m_{12}^2 = 7 \times 10^{-5} \text{eV}^2$, $\Delta m_{13}^2 = 2.5 \times 10^{-3} \text{eV}^2$, $\tan^2 \theta_{12} = 0.42$, $\tan^2 \theta_{23} = 1$, $\sin \theta_{13} = 0.1$, and $\delta = 0$.

2). The elements of the mass matrix are physical parameters: they can be immediately measured in the neutrinoless beta decay and, in principle, in other similar processes with $\Delta L = 2$.

3). In the SM and MSSM the radiative corrections (the renormalization group effects) on the mass matrix are very small, so they do not change the structure of the mass matrix up to the scale where the corresponding mass operators are formed or up to the symmetry scale.

The disadvantage is that the flavor basis may differ from the symmetry basis, where the symmetry (as well as mechanism of symmetry violation) and underlying dynamics are realized.

3.2. Observations

Scanning the $\rho - \sigma$ plots shown in Figs. 9, 10, and 11, one can make the following observations.

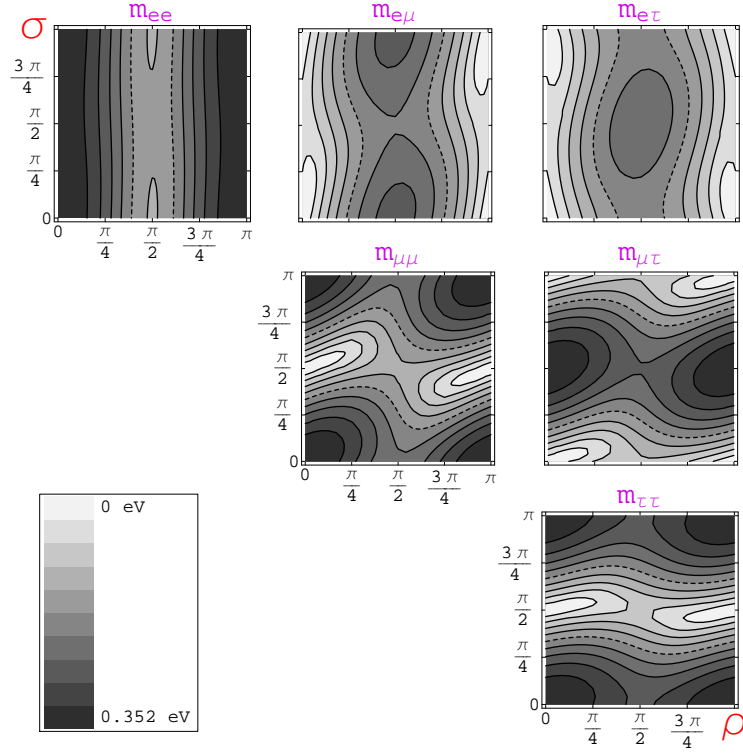


Figure 10: The same as in Fig. 9 for the quasi-degenerate spectrum: $m_3/m_2 = 1.01$, $m_1 \approx 0.35$ eV.

1). A large variety of different structures is still possible, depending strongly on the unknown m_1 , type of mass hierarchy and Majorana phases.

2). Generically the hierarchy of elements is not strong: within 1 order of magnitude. At the same time, matrices with one or two exact zeros are not excluded.⁴²⁾

3). Typically, the hierarchical structures appear for the Majorana phases near 0, $\pi/2$, or π .

4). Matrices are possible with:

- dominant (i) diagonal elements ($\sim I$), (ii) $\mu\tau$ -block, (iii) e -row elements, (iv) ee -, $\mu\tau$ -, $\tau\mu$ - elements (triangle structure),
- democratic structure,
- flavor alignment,
- non-hierarchical structures with all elements of the same order,
- flavor disordering, “anarchy”⁴³⁾.

5). Matrices can be parameterized in terms of powers of small parameter $\lambda = 0.2 - 0.3$ consistent with the Cabibbo mixing.

Clearly, at present there is enormous degeneracy in structures of the neutrino mass matrices which can reproduce data. The degeneracy will be reduced substantially,

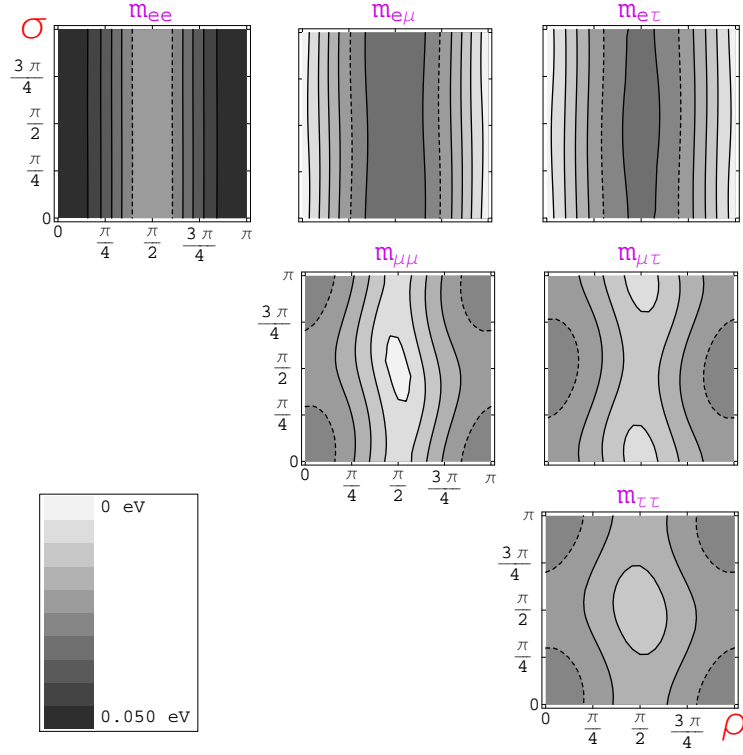


Figure 11: The same as in Fig. 9 for the inverted mass hierarchy: $m_3/m_2 = 0.1$, $m_3 \approx 0.005$ eV.

if we have more information about the absolute mass scale, the mass hierarchy and at least one Majorana phase.

4. New theoretical puzzle?

4.1. Expected and unexpected

What is behind the observed structure of the neutrino masses and mixing? Do we really encounter new theoretical problem? The hope was that neutrinos will reveal something simple which will shed a light on physics of the high energy scales and on the fermion masses in general.

A plausible scenario was:

- seesaw mechanism⁴⁴);
- quark-lepton symmetry, in a sense that $m_D(\text{neutrino}) \sim m(\text{quark})$;
- simple structure of M_R - the mass matrix of the RH neutrinos, *e.g.*, $M_R \propto I$ or $M_R \propto m(\text{quarks})$.

This leads typically to the hierarchical mass spectrum of light neutrinos and to small lepton mixing. If this scenario is confirmed, the problem would be probably closed, and it would be difficult to add something more.

Instead, unexpected pattern of the lepton mixing has been found with maximal or near maximal 2 - 3 mixing; large 1 - 2 mixing with, however, significant deviation from maximal mixing. Type of the spectrum is not yet clear. However hierarchy, if exists, is weaker than that for quarks and charged leptons.

All items of the “plausible scenario” are questioned now:

The quark-lepton symmetry? - Less obvious though, still can be realized at some level.

The seesaw mechanism? - Still is very appealing, though there is no clear indication from the pattern of mixing.

Simple structure of the mass matrix of the RH neutrinos? - Probably no.

At this point plenty of various scenarios have been suggested with two extremes which we will discuss next.

4.2. Two extremes: symmetry or no symmetry

I. Quasi-degenerate spectrum, maximal 2- 3 mixing. This certainly implies flavor symmetry like Z_2 , A_4 ⁴⁵⁾ or $SO(3)$ ⁴⁶⁾. Lepton (neutrino) mass/mixing pattern strongly deviates from quark masses and mixing pattern. Flavor symmetry of the neutrino sector is broken in the quark and charged lepton sectors. Oscillation observables Δm_{12}^2 and Δm_{23}^2 have no substantial imprint in the mass matrix: they appear as small perturbations of the structure determined by the mixing and the Majorana phases.

II. Hierarchical spectrum, deviation of the 2-3 mixing from maximal. No special symmetry is needed if 2-3 mixing deviates significantly from maximal value. In this case mixings can be determined by the condition of “naturalness” of the mass matrices according to which the mixing angles satisfy equalities

$$\tan \theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}, \quad (21)$$

where m_i are the eigenvalues ⁴⁷⁾.

Rotations which follow from diagonalizations of the up and down mass matrices cancel each other in the quark sector thus leading to a small quark mixing and they sum up in the leptonic sector ⁴⁷⁾. The later can be related somehow to the mechanism which leads to a smallness of the neutrino masses. In particular, for the 2-3 mixing we have

$$\theta_{23} \sim \sqrt{\frac{m_2}{m_3}} + \sqrt{\frac{m_\mu}{m_\tau}} \sim 38^\circ \quad (22)$$

which is well within the allowed region. In this case the oscillation parameters are well imprinted into the structure of mass matrix. The Majorana phases are not important. Notice that the present atmospheric neutrino data indeed give some hint of deviation of the 2-3 mixing from maximal as it was discussed in Sec. 2.2.

4.3. Large Mixing and Degeneracy

Is large mixing implies degeneracy of the neutrino spectrum? Or *vice versa* does degeneracy of the neutrino mass spectrum explains large or maximal mixing?

Let us consider the two generation case and introduce the degeneracy parameter

$$\delta_{23} \equiv \frac{\Delta m}{m} \approx \frac{\Delta m^2}{2m^2}, \quad (23)$$

as well as the parameter which characterizes the deviation of 2-3 mixing from maximal (7) Only in one case a strong degeneracy is related to small deviation from maximal mixing: this happens for the pseudo-Dirac neutrinos with mass matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & \epsilon \end{pmatrix}, \quad (24)$$

where $\epsilon \ll 1$. In this case $D_{23} \approx \delta_{23}$. For $m \sim 0.25$ eV, the atmospheric splitting implies $\delta_{23} \sim 0.03$. Therefore if $D_{23} > 0.01$ is established, the possibility of Eq. (24) will be excluded.

In other cases the deviation and the mass split are not related. For instance, the matrix

$$\begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \quad (25)$$

gives exactly maximal mixing $D_{23} = 0$ but arbitrary mass split: $\delta_{23} = 2\epsilon$. In contrast, the matrix

$$\begin{pmatrix} \epsilon & 1 \\ 1 & -\epsilon \end{pmatrix} \quad (26)$$

corresponds to zero mass split, $\delta_{23} = 0$, but arbitrary mixing $D_{23} = \epsilon/2$.

Simple relations between degeneracy and deviations is absent also due to the presence of

- third neutrino,
- Majorana phases.

Furthermore, in the case of the degenerate spectrum for the first and second generations, smaller splitting, $\delta_{12} \sim 10^{-3}$, is associated to larger deviation $D_{12} \sim 0.2$.

4.4. No simple structure?

Our attempts to find simple structures for the neutrino masses matrices may fail for the following reasons.

1). Neutrino mass matrix can obtain several relevant contributions from new physics at all possible scales, M_a , from the EW scale to the Planck scale. As a consequence, the structure of the mass matrix can be rather complicated. The effective operator at low energies (after integrating out the heavy degrees of freedom) can be written as⁴⁸⁾

$$\sum_a \frac{\lambda_{ij}^a}{M_a} (L_i H)^T (L_j H), \quad i, j = e, \mu, \tau, \quad (27)$$

where L_i is the lepton doublet, λ_{ij}^a are the dimensionless couplings. After the EW symmetry breaking it generates the neutrino masses

$$m_{ij} = \sum_a \frac{\lambda_{ij}^a \langle H \rangle^2}{M_a}. \quad (28)$$

The sum is crucial here. For $\lambda_{ij}^a \sim 1$ and $M = M_{Pl}$ we find $m_{ij} \sim 10^{-5}$ eV.⁴⁹⁾ Even this contribution can be relevant for the sub-leading structures of the mass matrix and phenomenology.

2). The presence of new (sterile) neutrino states can have dramatic consequences. Heavy sterile neutrinos ($M > 1$ keV) may contribute to the sum (28) modifying substantially mixing of the active neutrinos and mass splitting⁵⁰⁾. Light sterile neutrinos ($m < 1$ eV) can change low energy phenomenology and therefore determination of the neutrino (oscillation) parameters. Therefore to understand the neutrino masses and mixing we need to restrict or control possible effects of sterile neutrinos.

5. Conclusions

1. Enormous progress has been achieved in the determination of the neutrino masses and mixings, in reconstruction of the neutrino mass spectrum, and in studies of the neutrino mass matrix.

2. Achieved results allow us to formulate clear program of further experimental and phenomenological studies.

3. Amazing pattern of the lepton mixing emerges which probably composes the new theoretical puzzle. Its resolution may lead us to new fundamental results, to discovery of new symmetries of Nature.

4. From this theoretical perspective, the most important future measurements turn out to be

- determination of the absolute mass scale, and tests of degenerate scenario;
- searches for deviation of the 2-3 mixing from maximal value;

- measurements of 1-3 mixing, and of course,
- establishing the Majorana nature of neutrinos.

6. Acknowledgments

I am grateful to E. Kh. Akhmedov, P. de Holanda, M. Frigerio, C. Lunardini and O. Peres for fruitful discussions and contributions to this talk. Many thanks to Milla for this marvelous meeting.

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